PGIREM: Reliability-Constrained IR Drop Minimization and Electromigration Assessment of VLSI Power Grid Networks using Cooperative Coevolution

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***Abstract*—Due to the resistance of metal wires in power grid network, voltage drop noise occurs in the form of IR drop which may change the output logic of underlying circuits and may affect the reliability performance of a chip. Further, it is necessary to handle different reliability constraints while designing a robust power grid network for a chip. Any violation of such constraints may increase the occurrences of IR drop. Therefore, there is a need to minimize the IR drop without violating the reliability constraints. In this paper, the IR drop minimization problem is formulated as a single objective large-scale variable minimization problem subjected to different reliability constraints, such as IR drop constraints, electro-migration constraints, minimum width constraints, metal area constraints. At first, the large-scale minimization problem is divided into several subproblems using a divide and conquer based decomposition strategy, called Cooperative Coevolution. Secondly, each subproblem is solved using self-adaptive differential evolution with neighborhood search. Finally, electromigration (EM) assessment is done for the power grid networks using Black's equation to demonstrate the optimism in the predicted time-to-failure (TTF) after minimization of the IR drop.**

***Keywords*-Cooperative Coevolution, Electromigration, IR drop noise, Large Scale Optimization, Power Grid Network, Reliability-Constraints, VLSI.**

I. INTRODUCTION

Power grid networks of VLSI chip are becoming larger than ever with the scaling of technology node, which introduces the added design challenges and makes the design phase time-consuming and iterative. Generally, IR drop noise occurs in the power grid due to the metal resistance of the grid, which is one of the major concern while designing the power grid [1]. As the IR drop noise can change the voltage level of a logic block, hence it is essential to ensure the IR drop noise below a threshold level. The underlying logic block can malfunction if the IR drop noise exceeds a certain threshold level. Furthermore, EM-induced increase in metal resistances can also change the IR drop level [2]. Therefore, it is necessary to locate the IR drop noises and minimizing the IR drop noises occur in the power grid network.

The IR drop noises are located by doing power grid analysis of the whole network which is a process by which currents and voltages of the equivalent model of the power grid network are determined. In general, voltage drop noise violations are occurred by IR drop (due to the resistances of the metal lines) and Ldi/dt voltage drop (due to inductances of the metal lines and C4 bumps) noises. In this paper, we limit this work only for IR drop minimization based on metal width reduction. IR drop generally depends on the current flowing through the metal lines and the resistances of the metal lines. The amount of current flowing through the metal line is determined by the current drawn by the underlying functional blocks. Therefore, it is necessary to limit the flow of currents through the metal lines, to prevent any occurrences of EM. On the other hand, resistances of the metal lines depend on the width and length of the metal layer. If the length is kept constant, then by increasing the width of the metal layers the resistance of the metal lines can be reduced which will force the IR drop to decrease further.

In industry, there are many tools for IR drop analysis, such as $RedHawk^{TM}$, $PrimeRail^{TM}$, and $Totem^{TM}$. Therefore, using these tools power grid analysis is done and IR drop noises are located. Layout designers try to minimize the IR drop in the located area manually, by increasing the metal width (but confining within the design rules) of the power and ground lines, to decrease the resistances which reduce IR drop. After varying the widths of the metal lines, designers have to perform power grid analysis again to know whether the hotspots created by IR drop is below a certain threshold. And this process continues iteratively until the IR drop comes below the threshold level. As this is a time-consuming process, the manual design of power grid network after analysis becomes cumbersome. Also, no automated tools in the industry are available which can minimize IR drop in power grid network. Therefore, in this paper, we have constructed the IR drop minimization problem as a large-scale optimization problem. We also tried to propose a framework using Cooperative coevolution for minimizing the IR drop by varying the metal layer dimensions of power grid network which would help in automatically projecting metal widths within safer IR drop noise level.

To the best of our knowledge, this paper is the first to study the minimization of IR drop for power grid networks by changing the metal widths considering different reliability constraints. The major contribution of this paper includes:

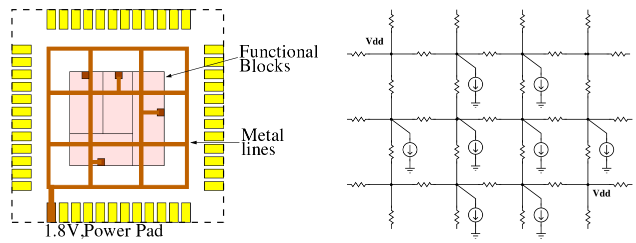
- The IR drop minimization problem has been formulated as the large-scale optimization problem for a simplified steady-state model of the power grid network.
- Cooperative coevolution based method is employed for the minimization of the IR drop of the power grid network.
- The proposed minimization approach is able to minimize the IR drop without changing the topology of the grid by only changing the metal widths.
- EM assessment of the power grid network is done for the optimized power grid with minimized IR drop which demonstrated optimism in the life time prediction of the chip.

The rest of the paper is arranged as follows. In Section II, the related work on power grid optimization is described. Power grid network model used in the paper is explained in Section III. Problem formulation and all the reliability constraints of the power grid network are described in Section IV. Section V describes the Cooperative Coevolution based approach and how it is implemented for doing IR drop minimization. The experimental results on different power grid benchmark circuits are mentioned in Section VI.

II. Related Work

There are several works on the power grid analysis and verification, such recent works are [3, 4]. The basic objective of the works done in power grid optimization so far is to minimize the area of the metal wires (considering IR drop as a constraint) of the power grid network by constructing two-phase optimization problem, then iteratively solve the two-phase optimization problems using different algorithms. Tan et al. [5] solved the problem using a sequence of linear programmings. Wang et al. [6] solved the same problem with sequential network simplex algorithm. Similarly, there are more works on the metal area minimization. Zeng et al. [7] have done power grid wire sizing optimization problem using locality driven partitioning based two-step optimization algorithm. Chang et al. [8] proposed routing friendly multilayer power grid network by allocating each layer metal width considering the impact of AP layer. However, there is hardly any work so far, which tried to minimize the IR drop by considering different power grid network constraints. Moreover, recent research developments are more concentrated on developing new physics-based EM models for power grids to achieve optimism in mean-time-to-failure (MTTF) [2, 9]. Minimizing IR drop can be one of the alternatives to obtain optimism in MTTF. In this paper, we tried to propose an approach to minimize the total IR drop of the whole power grid network by changing the metal widths of each of the metal segments (branches), which are manually done iteratively by the layout designers in the industry, to minimize the IR drop. Layout designers also use decoupling capacitances at few critical nodes to reduce the IR drop and Ldi/dt voltage noises. But in this paper, we are only considering IR drop minimization by varying metal widths. And here Cooperative Coevolution approach for solving large-scale optimization problem is employed.

III. Power Grid Network Model



(a) (b)

Figure 1. (a) An example of floor-plan and its power grid network(metal lines) with functional blocks (b) Modeling power lines to resistive network.

An illustration of floor-plan and its power grid network with functional blocks is shown in Figure 1(a). For minimization of IR drop voltage noise, steady-state model of the power grid network is considered in this paper, which is shown in Figure 1(b). In this model, only the resistance of the metal lines is considered. The current drawn by the underlying circuit is modeled as the current sinks connected to ground as shown in Figure 1(b). The vias of this model are considered having zero resistance, as vias have very low resistance. Inductances associated with the C4 bumps, to connect V_{dd} and ground connections are not considered here. Also, any other parasitic effects due to inductances and capacitances are not considered here. The steady-state model of the power grid network can be represented as a linear system of equations i.e., $\mathbf{GV} = \mathbf{I}$, where conductances of the metal lines make the $\mathbf{G}$ matrix, current sinks connected to ground contributes to form the $\mathbf{I}$ vector and node voltages of all the nodes generate the $\mathbf{V}$ vector. By using direct solvers, such as KLU solver [10], we can determine $\mathbf{V}$ i.e., voltages of all the nodes. Similarly, from node voltages of all the nodes, we can even find the branch currents.

IV. Problem Formulation

A. Objective Function for IR Drop Minimization

Let's consider $G = \{V, E\}$ be a graph corresponding to a power grid network, where $V = \{1, 2, \cdots, n\}$ is the set of all the n nodes of the power grid network and $E = \{1, 2, \cdots, b\}$ is the set of all the b branches of the graph corresponding to the steady state model of power grid network.

If I is the current passing through a metal segment (branch of the graph) of the Power Grid network having resistance R, then the voltage drop occurred across the metal segment can be represented by υ :

$$\upsilon = IR \tag{1}$$

With sheet resistance $\rho\ \Omega/\Box$ which is constant for a metal layer, having metal segment length and breadth of l and w respectively, the voltage drop can be denoted by the following:

$$\upsilon = I\frac{\rho l}{w}, \tag{2}$$

where $R = \frac{\rho l}{w}$ represents the total resistance of the metal segment of the power line. Similarly, the voltage drop of the whole Power Grid network with b number of metal wire segments (or branches) can be written as follows:

$$\begin{aligned} \upsilon &= \sum_{i=1}^{b} |I_i|\, R_i \\ &= \sum_{i=1}^{b} |I_i|\, \frac{\rho l_i}{w_i}, \end{aligned} \tag{3}$$

where $\mathbf{W}, \mathbf{I}, \mathbf{l}$ are set of vectors of metal widths, branch currents and metal lengths respectively i.e., $\mathbf{W} = (w_1, \cdots, w_b)$, $\mathbf{I} = (I_1, \cdots, I_b)$, $\mathbf{l} = (l_1, \cdots, l_b)$. For large value of b, equation (3) can be treated as the large scale optimization problem. In equation (3), I_i and w_i are the variables for the i^{th} metal segment which are non-separable in nature. Non-separable variables are those for which objective function depends on the interacting variables [11]. l_i has been taken as constant for the objective function throughout this work which will be imported from the circuit netlist. Therefore, for the whole power grid network, vectors $\mathbf{I}$ and $\mathbf{W}$ are sets of non-separable variables. Hence, the objective function can be formulated as a large-scale optimization problem with b non-separable variables as follows:

$$\upsilon(I_i, w_i) = \sum_{i=1}^{b} |I_i|\, \frac{\rho l_i}{w_i} \tag{4}$$

Hence the IR drop minimization problem with unequal branch currents I_i is given as follows:

$$\mathscr{P} : \underset{w_i \in \mathbf{W}\ \ I_i \in \mathbf{I}}{minimize}\ \upsilon, \tag{5}$$

subject to different reliability constraints which are described in section IV-B.

Theorem 1: Minimization of total IR drop υ of (4) reduces the worst case (maximum) IR drop.

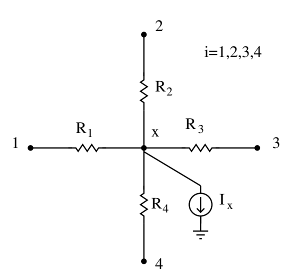


Figure 2. Single node of the power grid network model

Proof: The worst case maximum IR drop $\upsilon_{\text{IR max}}$ can be expressed as

$$\upsilon_{\text{IR max}} = Max(V_{DD} - V_x)\ \forall x \in V, \tag{6}$$

where V_x is the node voltage of the x^{th} node which depends on the voltage of the neighboring nodes and also depends on the current of the neighboring edges. Using KCL for a node V_i (see Figure 2), carrying current I_i from x to i, the expression of V_x can be written as follows:

$$\begin{aligned} I_i &= \frac{V_x - V_i}{R_i}\ \forall i \in K_s \\ \Rightarrow V_x &= V_i + I_i R_i \\ \Rightarrow V_x &= V_i + I_i \frac{\rho l_i}{w_i}, \end{aligned} \tag{7}$$

where K_s is the set of neighboring nodes of x. Therefore, V_x of (7) depends on the neighboring node voltages, neighboring branch currents and resistances. Also, υ of (3) depends on neighboring branch currents and resistances. Therefore, minimizing υ of (3) under the constraint $\mathcal{C}_1 : |I_{i\in E}|\, R_{i\in E} \leq \xi\ \ \forall i \in E$ by varying current and resistance will reduce the worst case maximum IR drop $\upsilon_{\text{IR max}}$ of (6). ■

B. Reliability Constraints of Power Grid Network

1) IR Drop Constraints: It can be defined by the following relation:

$$\mathcal{C}_1 : |I_{i\in E}|\, R_{i\in E} \leq \xi \tag{8}$$

The above relation should be maintained for all the i^{th} branches of the power grid network. ξ is the maximum tolerance level of voltage drop noise allowed between two consecutive nodes of the power grid network.

2) Metal Area Constraints: The metal area of the power grid network should be restricted to $\mathcal{A}_{max}$:

$$\mathcal{C}_2 : \sum_{i=1}^{b} l_i w_i \leq \mathcal{A}_{max} \tag{9}$$

3) Electromigration Constraints: To prevent the current carrying metal lines from electromigration, the current density of the metal lines should be less than I_m

$$\mathcal{C}_3 : \frac{I_{i\in E}}{w_{i\in E}} \leq I_m \tag{10}$$

4) Minimum Width Constraints: The minimum width of the metal lines w_{min} is limited by the technology on which the power grid network lies. Therefore, the constraint can be expressed as:

$$\mathcal{C}_4 : w_{i\in E} \geq w_{min} \tag{11}$$

5) KCL Constraints: KCL should be followed at all the n nodes of the power grid network.

$$\mathcal{C}_5 : \sum_{i=1}^{K} I_{j_i} + I_x = 0\ \forall j \in V \tag{12}$$

where K is the number of neighboring nodes of node j and I_x is the sink current of the model connected to ground.

V. Minimization using Cooperative Coevolution

A. Cooperative Coevolution(CC)

Cooperative Coevolution is a divide and conquer based approach to solve large scale variable optimization problems. It decomposes a large scale problem into several simple sub-problems. So the basic phenomenon of CC is that it decomposes an n-dimensional decision vector into n subcomponents and then optimizes each of the subcomponents using standard evolutionary optimization algorithm in a round robin fashion. The basic principle of the evolutionary optimization algorithm is to mimic the biological evolution process in generating good candidate solutions for a given objective function. Generally, candidate solutions of an optimization problem play the role of individuals in a population, and these individuals go under reproduction, mutation and recombination and selection to find the optimum solutions for a given objective function. Cooperative coevolution algorithm is stated in Algorithm 1.

Algorithm 1: Cooperative Coevolution Algorithm

Input: f, x_{min}, x_{max}, n
Output: Optimized value of f and corresponding variables $x_1, x_2, \cdots, x_n$ *values*

/*grouping based variable decomposition*/;
`groups` ← `grouping(`f, x_{min}, x_{max}, n`);`
/*Optimization stage using evolutionary algorithm*/;
`population` ← `rand(population_size,`n`);`;
for *j ← 1 to size(groups)* **do**
    `group_num ← groups[j];`
    `subpop ← population[:,group_num];`
    `subpop ← optimizer(best,subpop,FE);`
    `population[:,group_num] ← subpop;`
    `(best,best_val)←min(population);`

Theorem 2: CC based algorithm converges to global minimum for large-scale optimization problems if the main optimizer converges to the global minimum.

Proof: Let $f(x_1, x_2, \cdots, x_n)$ be an objective function with n decision variables. Now if n variables have been decomposed by random grouping with each group containing s decision variables, then $t = n/s$ number of subcomponents will be there. In other words, t instances of the objective function with each containing s number of decision variables and $(n - s)$ number of constants will be there. Now each of the t objective functions will be minimized independently using a standard optimization algorithm which will provide us with t local minimum values from t subcomponents. And then random grouping based strategy is applied to co-adapt these t minimum values. Hence, the global minimum will be obtained for the objective function f if the main optimization algorithm converges to the global minimum of each of the t instances of the objective function f. ■

CC is introduced into Genetic Algorithm for optimization of function by Potter et al. [12]. Liu et al. [13] used CC in large-scale optimization problem by using Fast Evolutionary Programming with Cooperative Coevolution. CC is introduced into PSO by Bergh et al. [14]. CC has also been adapted into Differential Evolution(DE) in [15], [11]. An improved version of DE is Self-Adaptive Differential Evolution with Neighborhood Search(SaNSDE)[16] which self-adapts its scaling factor F, crossover rate CR, and mutation strategy. It is proved that SaNSDE performs quite well compared to the other similar DE algorithms[16]. Yang et al. [11] showed that SaNSDE under CC framework (CC-SaNSDE) for large-scale variable optimization works very well. To deal with the non-separable nature of the problem, random grouping based decomposition strategy of the decision variables is used. Generally, in large-scale problems, only a proportion of variables interact with each other, therefore, the random grouping of variables increase the probability of grouping two interacting variables in the same subcomponent [17]. So CC-SaNSDE has been adapted in this paper to solve total IR drop minimization of power grid network.

B. IR drop minimization using CC-SaNSDE

Algorithm 2: IR drop minimization using CC-SaNSDE

Input: Both branch width & current ranges (for problem $\mathscr{P}$) are given as input from a power grid circuit netlist with b branches and n nodes. Branch lengths from the netlist are also taken as input.
Output: Optimum power grid netlist having minimized IR drop along-with the corresponding optimum resistance budget and metal width budget for the b branches.

Search space S is constructed in such a way to incorporate the reliability constraints $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$, $\mathcal{C}_4$, and $\mathcal{C}_5$ mentioned in section IV-B.;
while *inside search space* S **do**
    Initialize the initial parameters for CC-SaNSDE with random grouping.;
    Random grouping is employed to decompose the b variables in t subcomponents.;
    SaNSDE optimization algorithm is used to optimize each of the subcomponents.;
    Random grouping strategy is also used for the co-adaptation of all the subcomponents.;
Optimum metal widths corresponding to minimized IR drop is found and model parameters are updated.;
KLU solver is used to find the optimum IR drop.;

The IR drop minimization algorithm using CC-SaNSDE is given in Algorithm 2. For the problem $\mathscr{P}$, number of variables are decomposed to form subcomponents and then each of the subcomponents is minimized using SaNSDE. The subcomponents are formed based on random grouping of variables and each of the subcomponents are minimized using SaNSDE. And finally, random grouping based strategy is used for co-adaptation of the subcomponents. Also, SaNSDE has been modified to incorporate the reliability constraints as mentioned in section IV-B, to keep the search space within the region of validation. Branch widths are calculated corresponding to the resistances of the branches and ranges of branch width is given as input for the problem

$\mathscr{P}$. Apart from the branch width ranges, power grid analysis is done using KLU solver[10] to find the branch current ranges of the power grid network and given as input.

C. EM Assessment

EM assessment is done to predict the lifetime of a chip. EM occurs with two phases naming, nucleation phase, and the growth phase. In the nucleation phase, the voids started to form over a long period of time until the voids are nucleated. In the growth phase, hillocks are started to form and the interconnect metal resistance changes to a point where the resistance exceeds a threshold and failure occurs. The failure rates of the interconnect metal lines can be taken as a measure to check for EM-induced reliability. In extreme case, MTTF of the weakest metal segment can be treated as the life-time of the whole chip. Here, EM assessment of the power grid is done by considering Black's equation [18]. MTTF from Black's equation statistics can be written as follows:

$$MTTF = AJ^{-N} exp\{E_a/kT\}, \tag{13}$$

where A is constant which depends on the material properties of the metal. Here, k is the Boltzmann's constant and T is the temperature. Value of N is found to be 2, which depends on residual stress and current density J. E_a is the activation energy which also depends on current density J. The Black's equation of MTTF has been controversial and a better physics-based EM model is proposed in [2]. This proposed method [2] of predicting MTTF has also been used here to do the full chip life-time assessment of the power grid networks. In view of this, EM also causes the IR drop to increase as the resistance of the interconnect metal increases chronologically over a long period in the growth phase of EM. Therefore, minimizing the IR drop of the power grid network will surely demonstrate an optimism in the predicted MTTF.

VI. Experimental Results

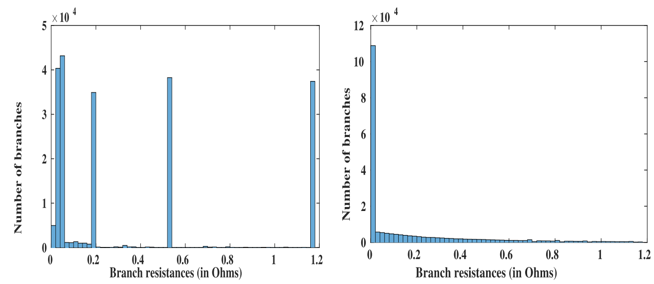
Table I
Power Grid Benchmark Circuits Data [19]

PG circuits	#Nodes(n)	#Branches(b)	Branch resistance ranges (in Ω)
ibmpg2	127238	208325	(0,1.17]
ibmpg3	851584	1401572	(0,9.36]
ibmpg4	953583	1560645	(0,2.34]
ibmpg5	1079310	1076848	(0,1.51]
ibmpg6	1670494	1649002	(0,17.16]
ibmpgnew1	1461036	2352355	(0,21.6]
ibmpgnew2	1461039	1422830	(0,21.6]

The CC-SaNSDE algorithm with random grouping is implemented in MATLAB and the experiments are performed on a machine with Intel Xeon E5-2650 processor having 32 GB memory and validated by IBM power grid benchmark

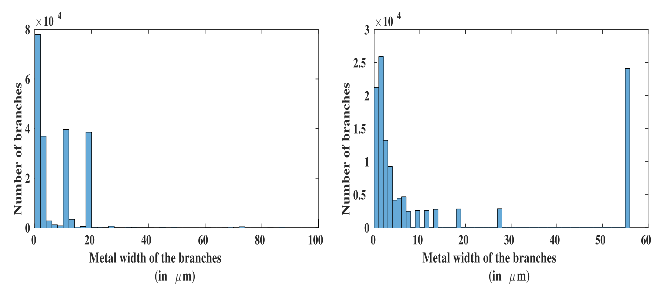
Table II
Comparision of IR drop for different Power Grid circuits before and after minimization

PG_circuits	IR drop (in volts)					
	Before minimization			After minimization		
	Max.	Avg.	Min	Max.	Avg.	Min.
ibmpg2	0.0631	0.0315	0.0129	0.0567	0.0283	0.0116
ibmpg3	0.0310	0.0252	0.0049	0.0279	0.0226	0.0044
ibmpg4	0.0386	0.0220	0.0060	0.0347	0.0198	0.0055
ibmpg5	0.0690	0.0373	0.0146	0.0621	0.0335	0.0134
ibmpg6	0.0598	0.0297	0.0111	0.0538	0.0267	0.0998
ibmpgnew1	0.0353	0.0204	0.0055	0.0317	0.0183	0.0051
ibmpgnew2	0.0516	0.0271	0.0102	0.0464	0.0243	0.0091



(a) (b)

Figure 3. Resistance budget for $ibmpg2$ circuit (a) before minimization, (b) after minimization.



(a) (b)

Figure 4. Metal width budget for $ibmpg2$ circuit (a) before minimization, (b) after minimization.

Table III
Comparision of MTTF for different Power Grid circuits using Balck's equation and using proposed method of [2] before and after minimization of IR drop.

PG_circuits	Mean time-to-failure (yrs)			
	Before minimization		After minimization	
	Black's	[2]	Black's	[2]
ibmpg2	7.81	15.65	11.21	19.35
ibmpg3	15.75	27.60	19.80	30.67
ibmpg4	12.55	29.25	16.33	32.42
ibmpg5	6.31	23.06	10.52	26.72
ibmpg6	9.49	17.75	13.34	21.10
ibmpgnew1	13.62	22.45	17.50	25.48
ibmpgnew2	12.41	20.10	16.22	23.37

circuits [19]. IBM power grid benchmark data for 7 circuits are listed in the Table I. Experiments are performed for these 7 circuits $i.e.$, $ibmpg2$ to $ibmpgnew2$. Current sink values of these circuits are modified so that the initial IR drop is below a threshold voltage level. Although metal width and length information are not available in the IBM power grid circuits, appropriate values of lengths are used and the corresponding width of metal layers are determined by considering sheet resistance of the metal $\rho = 0.02\ \Omega/\Box$ (assuming copper interconnect materials) and from branch resistances using equation (2). Algorithm 2 is tested using these power grid benchmarks and the obtained minimized IR drop data is listed in Table II. For the problem $\mathscr{P}$ power grid analysis is done using KLU solver [10] and all the branch currents are determined, from there branch current ranges have been found to be in the range of $0.1mA$ to $10mA$ for all the benchmark circuits. Subsequently, Algorithm 2 is used to get an optimum budget of metal widths and resistances corresponding to the minimum total IR drop of the power grid network. Figure 3 and Figure 4 show the resistance budget and metal width budget before and after minimization respectively for $ibmpg2$ circuit. Fitness Evaluation(FE) used for the experiments is 10^6 as for this value of FE the convergence of the Algorithm 2 is found to be the best.

For the EM assessment, we assume that the power grid will fail when the worst case IR drop exceeds $10\%V_{DD}$. In Black's equation based series model, the circuit is assumed to be failed as soon as any branch fails. Parameters used for calculation of MTTF of different power grid circuits in this paper are same as stated in [2]. Comparison of MTTF values for different power grid benchmarks before and after IR drop minimization for the Black's series model and the physics-based EM-model of [2] are listed in Table III. It is observed from the Table III that the MTTF after minimization of IR drop has increased significantly. That is because MTTF is inversely proportional to the current density raised to the power N (J^N). Since we have minimized IR drop by increasing the metal widths which decreases the J. As a result, we have got an optimistic prediction of the life-time of the power grid network with an increased value of MTTF.

VII. Conclusion

This paper presents a method to minimize the IR drop for power grid networks. The IR drop minimization problem for a power grid network is formulated as a large-scale optimization problem and the minimization of the IR drop is done using CC-SaNSDE. IR drop is minimized at the cost of the metal area of the chip. Results show minimization of IR drop for different power grid benchmarks. Further, EM assessment of the optimized power grid network is done to demonstrate the optimism in the life-time prediction of the power grid network.

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